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REZONE
A Proposal for Accomplishing Rezoning in Two-Dimensional Lagrangian Hydrodynamics Problems

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REZONE

A Proposal for Accomplishing Rezoning in Two-Dimensional Lagrangian Hydrodynamics Problems

by

Philip L. Browne

ABSTRACT

A proposed numerical experiment is described in which an attempt is made to provide a code which will rezone a two-dimensional Lagrangian hydrodynamics problem by allowing mass and the related momenta and internal energy to be transferred from one zone to another as the grid lines are moved through the material. Attempts to solve the distortion difficulties which occur in such problems by other methods have all pointed in the direction of rezoning as a desirable solution.

I. INTRODUCTION

For some time the two-dimensional Lagrangian hydrodynamics codes developed at Los Alamos have been used successfully on certain classes of problems in which the mesh at the start of the problem can be chosen in such a way that the distortions occurring in the zones during the problem are not too severe. In other classes of problems where the zone distortions become extreme, the problems become inaccurate and usually go beserk and become impossible to run, very often in the early stages.

For many years, rezoning of the mesh has been suggested as a possible remedy for these difficulties. But since two-dimensional codes and problems are already very complicated and tricky, the prospects have appeared rather hopeless and almost impossible; and the tendency has been to seek other means for preventing distortions. The author and a co-worker[1] are experimenting with such effects as viscosity, a pseudo-elasticity, and different forms of writing the pressure gradients in the momentum equation. The viscosity has helped a great deal in preventing certain types of distortions, but does not seem to do much good once the distortions become severe. The gradient study has aided our understanding, but not our results. The pseudo-elasticity study is just beginning. Others at Los Alamos have experimented with partial splitting, which again has helped in some cases but not in others.[2]

A tentative conclusion from all these projects is that most of these devices work well in a mesh which is not very distorted. However, in many problems, the distortions which occur must surely be real. Hence, it seems sensible that what is really required is some kind of rezoning or mixed Lagrangian-Eulerian calculation which will allow the transport of matter from one zone to another, to permit straightening of the mesh to remove distortions of the mesh but at the same time allowing distortions of the actual material to remain. The following discussion concerns a proposed model and a set of mechanical operations on that model which will allow computational experiments of this nature.

This proposal will be presented as a code which would be almost entirely independent of the hydrodynamics code itself. In other words, the hydrodynamics problem would be run for awhile with the pure Lagrangian code and then stopped when the mesh begins to get somewhat distorted. The REZONE code would then take over and smooth out the mesh, perhaps making several iterations; but there would be no time change during the rezoning operation. Then the mesh would be passed back to the hydrodynamics code for more time-dependent calculations. By this process of infrequent use of the rezoning, it is hoped that the mesh distortions may be held down enough to permit the problem to run satisfactorily. If this process appears to work it might even be possible to do the rezoning after every hydrodynamics cycle and thus produce a kind of mixed Lagrangian-Eulerian code.

II. THE MODEL

The general model adopted here is that of a mesh consisting of quadrilateral zones imbedded in the material to be studied.[3] A system of cylindrical symmetry is assumed and is described in an (r,z) coordinate frame. Figure 1 illustrates a typical mesh point and the four adjacent zones.

The notation required in the rezoning problem will be very complicated, and thus far no clearly superior system has been discovered for doing it neatly. Therefore, in order to reduce possible confusion on our part, we will adopt the notation and conventions used in our other projects.[1,3] The general convention is to number every thing in a clockwise direction as one goes around a point or zone. (If the r,z axes are interchanged - i.e., a mirror reflection about the 45° line

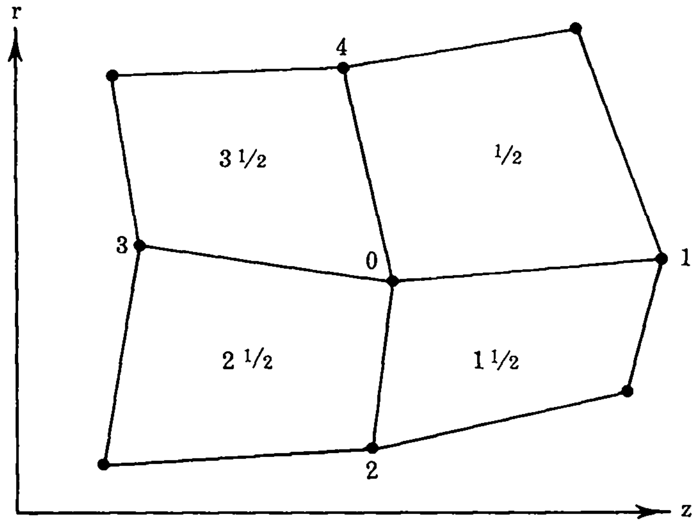

Figure 1

is made - so that one goes around points or zones in a counter clockwise manner, the same formulae will hold.)

Referring to Fig. 1, integral subscripts (1, 2, 3, and 4) will be used to denote quantities associated with the points. For example,

the coordinates are r_0, z_0; r_1, z_1; etc. (1a)

and the velocity components are $\dot{r}_0$, $\dot{z}_0$; $\dot{r}_1$, $\dot{z}_1$; etc. (1b)

Fractional subscripts will be used to denote quantites associated with the zones. For example,

$$\left.\begin{array}{l} \text{the relative volumes } V_{1\frac{1}{2}} = \dfrac{\mathcal{V}_{1\frac{1}{2}}}{\mathcal{V}_{0,1\frac{1}{2}}}, \text{ etc.,} \\ \text{where} \\ \mathcal{V}_{1\frac{1}{2}} = \text{present actual volume of the zone, and} \\ \mathcal{V}_{0,1\frac{1}{2}} = \text{original actual volume of the zone;} \end{array}\right\} \qquad (2a)$$

the pressure and Richtmyer-Von Neumann artificial dissipative term $(p+q)_{1\frac{1}{2}} = P_{1\frac{1}{2}}$ etc.; (2b) and the internal energy per unit original volume

$$\left.\begin{aligned} &\epsilon_{1\frac{1}{2}} = E_{1\frac{1}{2}}\, \rho_{0,1\frac{1}{2}} \quad \text{etc.,} \\ &\text{where} \\ &E_{1\frac{1}{2}} = \text{internal energy per unit mass, and} \\ &\rho_{0,1\frac{1}{2}} = \text{original mass density.} \end{aligned}\right\} \qquad (2c)$$

In our other studies[1] it was found to be most useful to adopt a model in which each zone was thought of as divided into four subzones (Fig. 2) obtained by joining a point, 8, within the zone (see Section IV) to the midpoints of the four sides. Quantities related to the four subzones are denoted by superscripts 0, 2, 4, and 6, numbered clockwise around the zone, starting at the vertex. In our experience, it has been most helpful to consider the masses of these subzones to be associated with the adjacent vertices for velocity, momentum (and kinetic energy) calculations, but to consider them related to the zone in which they lie for internal energy (and pressure calculations). For example, in Fig. 2, the masses $m^0_{\frac{1}{2}}$, $m^0_{1\frac{1}{2}}$, $m^0_{2\frac{1}{2}}$, and $m^0_{3\frac{1}{2}}$ are assumed to have the velocities $\dot{r}_0$ and $\dot{z}_0$ of the vertex, 0, etc...., while the masses $m^0_{\frac{1}{2}}$, $m^2_{\frac{1}{2}}$, $m^4_{\frac{1}{2}}$, and $m^6_{\frac{1}{2}}$ are assumed to have the internal energy $\epsilon_{\frac{1}{2}}$ of the zone, $\frac{1}{2}$, etc.

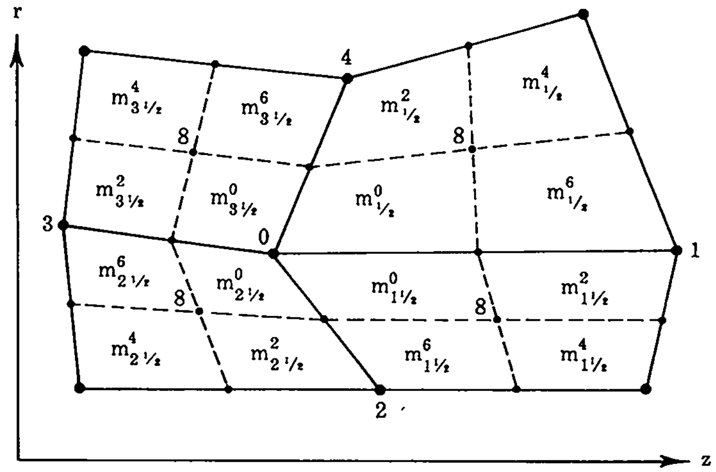

Figure 2

III. THE REZONING CODE

For purposes of clarity in description and coding, it seems best to organize the rezoning code into several passes through the mesh, which will be called:

First pass: The displacement pass
Second pass: The vertex pass
Third pass: The midpoint pass (3)
Fourth pass: The point 8 pass
Fifth pass: The averaging pass

For each vertex $(i, j \sim 0)$ and the related zone $(i + \frac{1}{2}, j + \frac{1}{2} \sim 1\frac{1}{2})$ and its subzones, we will need storage for the following quantities:

The "old" coordinates: r_0, z_0
The subzone masses: $m^0_{1\frac{1}{2}}$, $m^2_{1\frac{1}{2}}$, $m^4_{1\frac{1}{2}}$, $m^6_{1\frac{1}{2}}$
The point 8 coordinates: $r_{8,1\frac{1}{2}}$, $z_{8,1\frac{1}{2}}$
The actual subzone volumes: $\mathcal{V}^0_{1\frac{1}{2}}$, $\mathcal{V}^2_{1\frac{1}{2}}$, $\mathcal{V}^4_{1\frac{1}{2}}$, $\mathcal{V}^6_{1\frac{1}{2}}$ (4)
The actual subzone energies: $\mathcal{E}^0_{1\frac{1}{2}}$, $\mathcal{E}^2_{1\frac{1}{2}}$, $\mathcal{E}^4_{1\frac{1}{2}}$, $\mathcal{E}^6_{1\frac{1}{2}}$
The subzone r velocities: $\dot{r}^0_{1\frac{1}{2}}$, $\dot{r}^2_{1\frac{1}{2}}$, $\dot{r}^4_{1\frac{1}{2}}$, $\dot{r}^6_{1\frac{1}{2}}$
The subzone z velocities: $\dot{z}^0_{1\frac{1}{2}}$, $\dot{z}^2_{1\frac{1}{2}}$, $\dot{z}^4_{1\frac{1}{2}}$, $\dot{z}^6_{1\frac{1}{2}}$
The "new" coordinates: r^*_0, z^*_0

Brief descriptions of these passes and quantities are given in the following sections.

IV. THE DISPLACEMENT PASS

The purpose of this pass is to obtain the quantities listed in (4) and store them as data for the rezoning code. Some are obtained easily, and others will require calculation and perhaps some experimentation.

Quantities which can be obtained directly from the hydrodynamics data storage are the "old" coordinates, the subzone masses, and the subzone velocities. The calculation of point 8 may require some experimentation.

To do the rezoning exactly, which the author thinks is preferable, would require that one write the four subzone volumes in terms of the two unknowns r_8 and z_8 and set up four equations in these two unknowns,

$$
\begin{aligned}
\mathcal{V}^{0}_{1\frac{1}{2}} &= m^{0}_{1\frac{1}{2}}/\rho_{1\frac{1}{2}} & \qquad \mathcal{V}^{2}_{1\frac{1}{2}} &= m^{2}_{1\frac{1}{2}}/\rho_{1\frac{1}{2}} \\
\mathcal{V}^{4}_{1\frac{1}{2}} &= m^{4}_{1\frac{1}{2}}/\rho_{1\frac{1}{2}} & \qquad \mathcal{V}^{6}_{1\frac{1}{2}} &= m^{6}_{1\frac{1}{2}}/\rho_{1\frac{1}{2}}
\end{aligned}
\qquad (5)
$$

where $\rho_{1\frac{1}{2}} = (\rho_0/V)_{1\frac{1}{2}}$ is the uniform density of the whole zone, available from the hydrodynamic data storage. It is felt that except possibly for very distorted zones (which this code is intended to prevent), these four equations will have a solution for r_8 and z_8. With this method the four subzone volumes can be obtained from the simple relations, Eqs. (5).

Other simpler but less accurate methods are possible. For example, point 8 could be taken as the average of the four corners of the zone. This would require the volumes, $\mathcal{V}$, to be calculated from the coordinates of the corners of the zone and point 8. This in turn would mean that each subzone has a different density, $\rho^{0}_{1\frac{1}{2}} = m^{0}_{1\frac{1}{2}}/\mathcal{V}_{1\frac{1}{2}}$, etc.

Regardless of the method used for point 8, the energies will be given by

$$
\begin{aligned}
\mathcal{E}^{0}_{1\frac{1}{2}} &= E_{1\frac{1}{2}}\, m^{0}_{1\frac{1}{2}} & \qquad \mathcal{E}^{2}_{1\frac{1}{2}} &= E_{1\frac{1}{2}}\, m^{2}_{1\frac{1}{2}} \\
\mathcal{E}^{4}_{1\frac{1}{2}} &= E_{1\frac{1}{2}}\, m^{4}_{1\frac{1}{2}} & \qquad \mathcal{E}^{6}_{1\frac{1}{2}} &= E_{1\frac{1}{2}}\, m^{6}_{1\frac{1}{2}}
\end{aligned}
\qquad (6)
$$

where

$$
\begin{aligned}
E_{1\frac{1}{2}} &= \text{energy per unit mass of the zone} \\
&= (\epsilon/\rho_0)_{1\frac{1}{2}}
\end{aligned}
$$

The remaining quantities to be calculated are the "new" coordinates, r_0^* and z_0^*. The calculation of these coordinates is going to require considerable thought and experimentation. The basic objective of this process is to displace some or all of the vertices of the mesh in such a way as to reduce the distortions in the mesh. For example, in the simple case shown in Fig. 3, it appears that if the vertex, 0, were to be displaced to some point like 0*, the mesh would be less distorted.

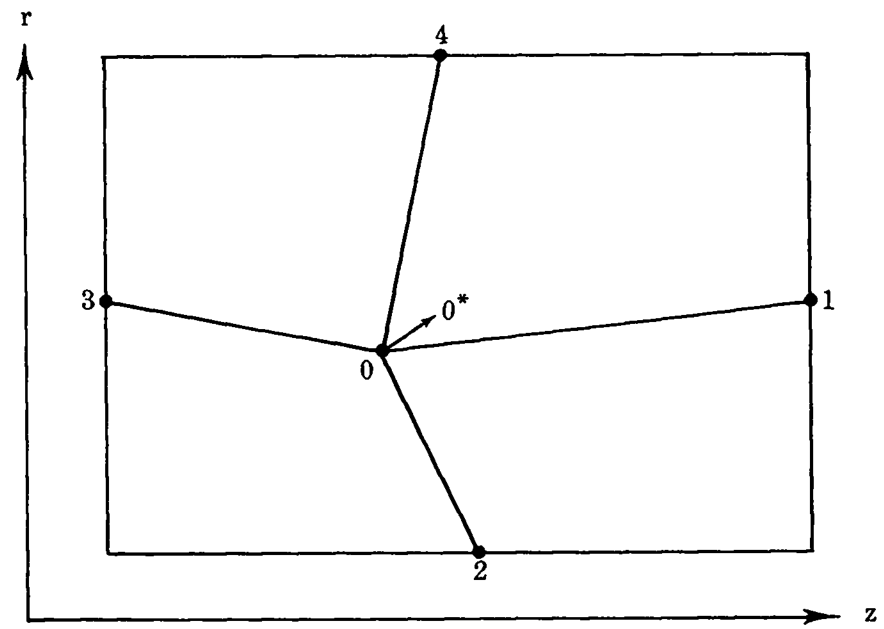

Figure 3.

There immediately comes to mind a number of possible ways in which the displacement $\overrightarrow{00^*}$ could be determined. These seem to fall into two main categories. Into one category we can classify methods which are directed primarily at equalizing angles about a vertex, and into the other category would go methods which tend to equalize both angles and distances between points. At present, the author feels that the equalization of angles is the preferable objective, because in a Lagrangian problem the spacing is often nonuniform for good reasons, such as the presence of a shock, the need for finer detail in one region of a problem than another, etc.

As an example of the first category, consider a system of four vectors of equal magnitude, each vector lying along a side coming into the vertex, O (Fig. 4). The resultant of these four vectors will give a displacement which should be in the proper direction to equalize the angles at the vertex. It is not quite so obvious as to how the magnitude should be chosen.

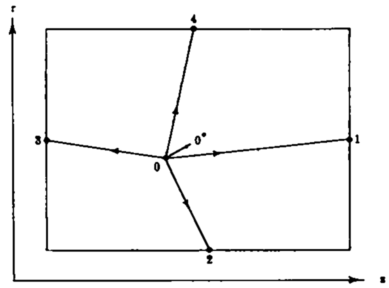

Figure 4.

As an example of the second category, consider a system of four vectors directed along the four sides, each vector of a magnitude which is a function of the length of the side. If the magnitude of the vector is chosen proportional to the length of the side, the resultant would have a magnitude and orientation which would tend to equalize both the angles and the spacing.

Another possibility is to consider O* to be the intersection of two lines joining the midpoints of opposite sides.

It is hoped at some later time to look at these suggested methods both theoretically and experimentally with the hope of finding ways to choose between various methods for calculating the displacement $\overrightarrow{OO^*}$. Our present leanings are toward the angular method described in connection with Fig. 4.

V. THE VERTEX PASS

The purpose of the next three passes is to shift the mesh through the fluid while keeping exact account of the volume, mass, internal energy, and momenta which will be transferred between subzones in the mesh. We wish to emphasize that in changing the mesh we are changing the subzones with which various mass, momenta, and energy are associated, but we are not moving the fluid. The vertex pass is the first step of this process; and as the name implies, we are working at the vertices.

Consider the simpler example used in Fig. 3 with the various subzones sketched in Fig. 5.

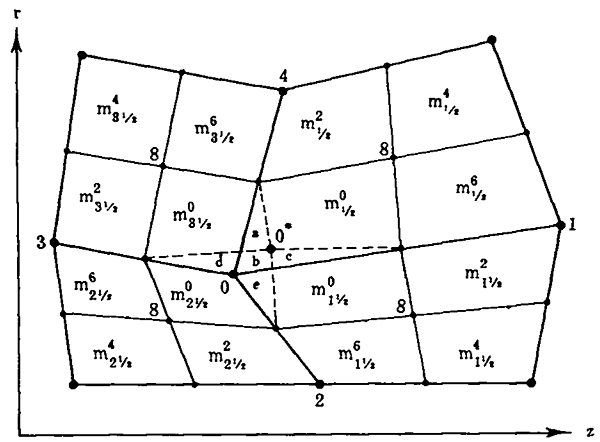

Figure 5.

If the new vertex, 0*, obtained in the displacement pass, is joined to the midpoints of the sides which meet at 0, a number of triangles and quadrilaterals are defined. Denote the volumes of these elements, which can be calculated exactly, by

$$\Delta \mathcal{V}_a,\ \Delta \mathcal{V}_b,\ \Delta \mathcal{V}_c,\ \Delta \mathcal{V}_d,\ \text{and}\ \Delta \mathcal{V}_e \tag{7a}$$

If the vertex 0 is shifted to 0* with the midpoints of the sides held constant, the volumes $\Delta \mathcal{V}$ will be shifted from one subzone to another. For example $\Delta \mathcal{V}_a$ will be shifted from subzone (½,0) to subzone (3½,0), etc. On this pass all transfers will be among the four subzones: (½,0), (1½,0), (2½,0), and (3½,0).

It will now be possible to write equations conserving volume, mass, internal energy, and r and z momentum.

The equations for volume transfer will be (denote volumes before transfer by $\mathcal{V}^-$ and volumes after transfer by $\mathcal{V}^+$)

$$
\begin{aligned}
(\mathcal{V}^0_{\frac{1}{2}})^+ &= (\mathcal{V}^0_{\frac{1}{2}})^- - \Delta\mathcal{V}_a - \Delta\mathcal{V}_b - \Delta\mathcal{V}_c \\
(\mathcal{V}^0_{1\frac{1}{2}})^+ &= (\mathcal{V}^0_{1\frac{1}{2}})^- + \Delta\mathcal{V}_c - \Delta\mathcal{V}_e \\
(\mathcal{V}^0_{2\frac{1}{2}})^+ &= (\mathcal{V}^0_{2\frac{1}{2}})^- + \Delta\mathcal{V}_b + \Delta\mathcal{V}_d + \Delta\mathcal{V}_e \\
(\mathcal{V}^0_{3\frac{1}{2}})^+ &= (\mathcal{V}^0_{3\frac{1}{2}})^- + \Delta\mathcal{V}_a - \Delta\mathcal{V}_d
\end{aligned}
\qquad (7b)
$$

The corresponding masses transferred will be given by

$$
\Delta m_a = \left(\frac{m^0_{\frac{1}{2}}}{\mathcal{V}^0_{\frac{1}{2}}}\right)^- \Delta\mathcal{V}_a \qquad \Delta m_b = \left(\frac{m^0_{\frac{1}{2}}}{\mathcal{V}^0_{\frac{1}{2}}}\right)^- \Delta\mathcal{V}_b
$$

$$
\Delta m_c = \left(\frac{m^0_{\frac{1}{2}}}{\mathcal{V}^0_{\frac{1}{2}}}\right)^- \Delta\mathcal{V}_c \qquad \Delta m_d = \left(\frac{m^0_{3\frac{1}{2}}}{\mathcal{V}^0_{3\frac{1}{2}}}\right)^- \Delta\mathcal{V}_d \qquad (8a)
$$

$$
\Delta m_e = \left(\frac{m^0_{1\frac{1}{2}}}{\mathcal{V}^0_{1\frac{1}{2}}}\right)^- \Delta\mathcal{V}_e
$$

and the corresponding mass transfer equations

$$
\begin{aligned}
(m^0_{\frac{1}{2}})^+ &= (m^0_{\frac{1}{2}})^- - \Delta m_a - \Delta m_b - \Delta m_c \\
(m^0_{1\frac{1}{2}})^+ &= (m^0_{1\frac{1}{2}})^- + \Delta m_c - \Delta m_e \\
(m^0_{2\frac{1}{2}})^+ &= (m^0_{2\frac{1}{2}})^- + \Delta m_b + \Delta m_d + \Delta m_e \\
(m^0_{3\frac{1}{2}})^+ &= (m^0_{3\frac{1}{2}})^- + \Delta m_a - \Delta m_d
\end{aligned}
\qquad (8b)
$$

In the case of an interface between two subzones it will be necessary to set the corresponding

$$\Delta m = 0 \tag{9}$$

in order to prevent mixing of different materials in a zone. One could still allow ΔV transfer and possibly energy transfer in order to straighten up the mesh.

The energies transferred will be given by

$$\Delta \mathcal{E}_a = \left(\frac{\mathcal{E}^0_{1\frac{1}{2}}}{m^0_{\frac{1}{2}}}\right)^- \Delta m_a \qquad \Delta \mathcal{E}_b = \left(\frac{\mathcal{E}^0_{\frac{1}{2}}}{m^0_{\frac{1}{2}}}\right)^- \Delta m_b$$

$$\Delta \mathcal{E}_c = \left(\frac{\mathcal{E}^0_{\frac{1}{2}}}{m^0_{\frac{1}{2}}}\right)^- \Delta m_c \qquad \Delta \mathcal{E}_d = \left(\frac{\mathcal{E}^0_{3\frac{1}{2}}}{m^0_{3\frac{1}{2}}}\right)^- \Delta m_d \tag{10a}$$

$$\Delta \mathcal{E}_e = \left(\frac{\mathcal{E}^0_{1\frac{1}{2}}}{m^0_{1\frac{1}{2}}}\right)^- \Delta m_e$$

and the corresponding energy transfer equations

$$(\mathcal{E}^0_{\frac{1}{2}})^+ = (\mathcal{E}^0_{\frac{1}{2}})^- - \Delta\mathcal{E}_a - \Delta\mathcal{E}_b - \Delta\mathcal{E}_c$$

$$(\mathcal{E}^0_{1\frac{1}{2}})^+ = (\mathcal{E}^0_{1\frac{1}{2}})^- + \Delta\mathcal{E}_c - \Delta\mathcal{E}_e \tag{10b}$$

$$(\mathcal{E}^0_{2\frac{1}{2}})^+ = (\mathcal{E}^0_{2\frac{1}{2}})^- + \Delta\mathcal{E}_b + \Delta\mathcal{E}_d + \Delta\mathcal{E}_e$$

$$(\mathcal{E}^0_{3\frac{1}{2}})^+ = (\mathcal{E}^0_{3\frac{1}{2}})^- + \Delta\mathcal{E}_a - \Delta\mathcal{E}_d$$

The calculation of momentum transfer is really unnecessary on the vertex pass, because all the subzones involved in this pass have the same velocities. However, for completeness of description the momentum equations will be described because they will be needed on the later passes.

Assuming that when a given mass Δm is transferred out of a subzone, the corresponding momentum losses are $\dot{r}\Delta m$ and $\dot{z}\Delta m$, where $\dot{r}$ and $\dot{z}$ are the velocities of that subzone, we can write the conservation of r momentum equations

$$
\begin{aligned}
(m^0_{\frac{1}{2}})^+(\dot{r}^0_{\frac{1}{2}})^+ &= (m^0_{\frac{1}{2}})^-(\dot{r}^0_{\frac{1}{2}})^- - (\Delta m_a)(\dot{r}^0_{\frac{1}{2}})^- - (\Delta m_b)(\dot{r}^0_{\frac{1}{2}})^- - (\Delta m_c)(\dot{r}^0_{\frac{1}{2}})^- \\
(m^0_{1\frac{1}{2}})^+(\dot{r}^0_{1\frac{1}{2}})^+ &= (m^0_{1\frac{1}{2}})^-(\dot{r}^0_{1\frac{1}{2}})^- + (\Delta m_c)(\dot{r}^0_{\frac{1}{2}})^- - (\Delta m_e)(\dot{r}^0_{1\frac{1}{2}})^- \\
(m^0_{2\frac{1}{2}})^+(\dot{r}^0_{2\frac{1}{2}})^+ &= (m^0_{2\frac{1}{2}})^-(\dot{r}^0_{2\frac{1}{2}})^- + (\Delta m_b)(\dot{r}^0_{\frac{1}{2}})^- + (\Delta m_d)(\dot{r}^0_{3\frac{1}{2}})^- + (\Delta m_e)(\dot{r}^0_{1\frac{1}{2}})^- \\
(m^0_{3\frac{1}{2}})^+(\dot{r}^0_{3\frac{1}{2}})^+ &= (m^0_{3\frac{1}{2}})^-(\dot{r}^0_{3\frac{1}{2}})^- + (\Delta m_a)(\dot{r}^0_{\frac{1}{2}})^- - (\Delta m_d)(\dot{r}^0_{3\frac{1}{2}})^-
\end{aligned}
\qquad (11)
$$

which can be solved for the new $\dot{r}^+$ using the results of (8b). There will be a corresponding set of conservation of z momentum equations with $\dot{r}$ replaced by $\dot{z}$.

This completes the calculations for the vertex pass. In this description of the calculations we have considered only the simple case shown in Fig. 5. There are a number of other possible cases depending on the relative orientations of $\overrightarrow{OO^*}$ and the sides. These cases and the corresponding equations will be included in a later report.

VI. THE MIDPOINT PASS

After completion of the vertex pass, the starting mesh of Fig. 5 will take on a shape of the nature represented in Fig. 6. The lines joining the new vertices will no longer be straight lines, but will consist of two line segments joining the new vertices to the old midpoints. In storage for each subzone there will be values of $\mathcal{V}$, m, $\mathcal{E}$, $\dot{r}$, and $\dot{z}$ after the vertex pass.

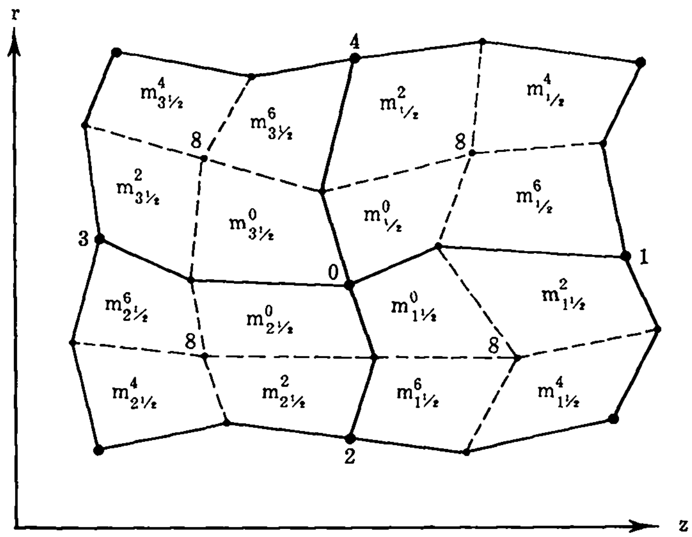

Figure 6.

The purpose of the midpoint pass is to straighten out the lines between vertices (move "old" midpoints to "new" midpoints) and at the same time accomplish the exact transfer of volume, mass, energy, and momentum between subzones. It is felt that with proper arrangement of the data available from the storage in the REZONE code, the same transfer code as was used in the vertex pass can be used here.

For example, consider the line between 0 and 1 in Fig. 6 and the four adjacent subzones (Fig. 7). If the "old" midpoint M is shifted to

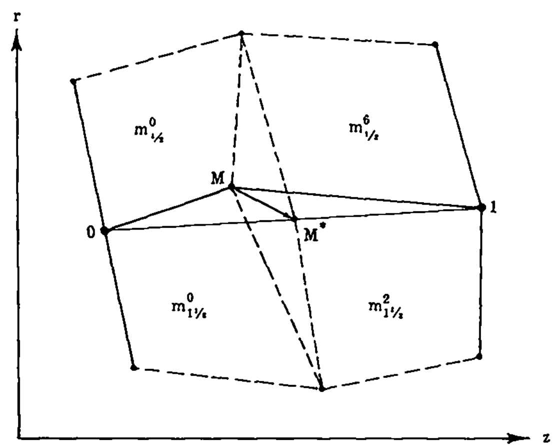

Figure 7.

the "new" midpoint M*, it is apparent that we have exactly the same situation as shown in Fig. 5 for the displacement of the vertex 0 to 0*, except that the notation for the subzones is different. Consequently, the same equations for calculation of transfer of volume, mass, internal energy, and momenta can be used provided the proper notation for the subzones is substituted. Here again there will be other possible cases than the one shown, but a code written to cover all possible cases in the displacement pass would cover all possible cases here.

VII. THE POINT 8 PASS

After the midpoint pass, the mesh will again consist of vertices and straight sides between the vertices, hopefully with less distortion than in the original mesh. However there is a possibility that the two previous passes may have concentrated large (or small) amounts of mass, energy, or momentum in some of the subzones. It seems wise, therefore, to include a pass in which point 8 is shifted to a new position and the corresponding transfer of volume, mass, energy, and momenta are made.

Consider a typical zone after the midpoint pass (Fig. 8). The "old"

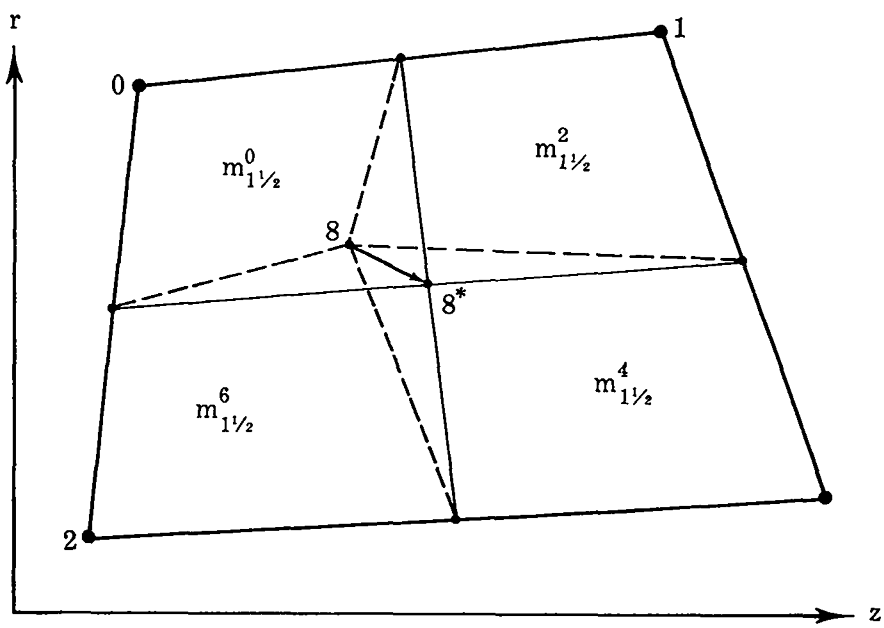

Figure 8.

point 8 is now joined to the new midpoints to define the four subzones. In storage are listed the V, m, $\mathcal{E}$, $\dot{r}$, and $\dot{z}$ as calculated after the midpoint pass.

Now, it is necessary to define a new point 8*. Again, a number of possibilities are apparent. Any of the devices suggested for calculating the displacement could be considered. Another possibility would be to calculate 8* as the average of the four corners of the zone.

Once again, we see that the situation is exactly the same as for the displacement of 0 to 0*, except for the notation of the subzones. Thus the same equations of transfer may be used.

VIII. THE AVERAGING PASS

At the completion of the point 8 pass, the mesh has been completely redefined with new positions for the vertices, midpoints of the sides, and point 8's interior to the zones. However, the subzones are still represented in the storage as separate entities. In order to return to the hydrodynamics code, it is necessary to average certain quantities to fit them back into the model used for the hydrodynamics code.

For the vertices the new velocities are needed. These are obtained by dividing the total momentum of the four subzones around the vertex by the sum of the four masses,

$$\dot{r}_0 = \frac{m^0_{\frac{1}{2}} \dot{r}^0_{\frac{1}{2}} + m^0_{1\frac{1}{2}} \dot{r}^0_{1\frac{1}{2}} + m^0_{2\frac{1}{2}} \dot{r}^0_{2\frac{1}{2}} + m^0_{3\frac{1}{2}} \dot{r}^0_{3\frac{1}{2}}}{m^0_{\frac{1}{2}} + m^0_{1\frac{1}{2}} + m^0_{2\frac{1}{2}} + m^0_{3\frac{1}{2}}} \tag{12}$$

There is a similar equation involving $\dot{z}$.

For the zones, a new V, ϵ, and (p+q) are required. For the relative volume and energy,

$$V_{1\frac{1}{2}} = \left(\frac{\rho_0}{\rho}\right)_{1\frac{1}{2}} = (\rho_0)_{1\frac{1}{2}} \left[\frac{V^0_{1\frac{1}{2}} + V^2_{1\frac{1}{2}} + V^4_{1\frac{1}{2}} + V^6_{1\frac{1}{2}}}{m^0_{1\frac{1}{2}} + m^2_{1\frac{1}{2}} + m^4_{1\frac{1}{2}} + m^6_{1\frac{1}{2}}}\right] \tag{13}$$

$$\epsilon_{1\frac{1}{2}} = (\rho_0 E)_{1\frac{1}{2}} = (\rho_0)_{1\frac{1}{2}} \frac{\mathcal{E}^0_{1\frac{1}{2}} + \mathcal{E}^2_{1\frac{1}{2}} + \mathcal{E}^4_{1\frac{1}{2}} + \mathcal{E}^6_{1\frac{1}{2}}}{m^0_{1\frac{1}{2}} + m^2_{1\frac{1}{2}} + m^4_{1\frac{1}{2}} + m^6_{1\frac{1}{2}}} \qquad (14)$$

The new pressure, p, can be obtained from the equation of state

$$p = p(V,\epsilon) \qquad (15)$$

The Richtmyer-Von Neumann q term presents a different problem. A simple but somewhat inaccurate approximation would be to use the same q term that existed in a zone before the rezoning, but there are two other methods that seem preferable.

One alternative would be to recalculate a q term for each new zone. This could be done by various methods; but in order to be consistent with the present hydrodynamics code, it would seem best to express the volume of the zone in terms of the coordinates of the corners

$$\mathcal{V}(r, \ldots, z, \ldots)$$

and then differentiate with respect to time to obtain $d\mathcal{V}/dt$ in terms of the velocities and coordinates of the corners of the zone. The new q could then be calculated in terms of $(1/\mathcal{V})(d\mathcal{V}/dt)$ from the same expression used in the hydrodynamics code.

The third alternative would be to treat q as a separate quantity of unit energy/volume = force/area, and to transfer it along with volume as the various passes are made. Such a procedure would require four additional storages per zone in which would be stored for each subzone an actual or total Q given by

$$Q^0_{1\frac{1}{2}} = q_{1\frac{1}{2}} \mathcal{V}^0_{1\frac{1}{2}}, \text{ etc.} \qquad (16)$$

One would then need to do calculations which conserve Q, by transferring ΔQ from one subzone to another, just as was done for $\mathcal{V}$, m, $\mathcal{E}$, etc.

Thus, following Eqs. (10b) there would be a set of equations defining

$$\Delta Q_a = \left(\frac{Q^0_{\frac{1}{2}}}{\mathcal{V}^0_{\frac{1}{2}}}\right)^- \Delta\mathcal{V}_a \qquad \Delta Q_b = \left(\frac{Q^0_{\frac{1}{2}}}{\mathcal{V}^0_{\frac{1}{2}}}\right)^- \Delta\mathcal{V}_b$$

$$\Delta Q_c = \left(\frac{Q^0_{\frac{1}{2}}}{\mathcal{V}^0_{\frac{1}{2}}}\right)^- \Delta\mathcal{V}_c \qquad \Delta Q_d = \left(\frac{Q^0_{3\frac{1}{2}}}{\mathcal{V}^0_{3\frac{1}{2}}}\right)^- \Delta\mathcal{V}_d \tag{17}$$

$$\Delta Q_e = \left(\frac{Q^0_{1\frac{1}{2}}}{\mathcal{V}^0_{1\frac{1}{2}}}\right)^- \Delta\mathcal{V}_e$$

and the corresponding transfer equations

$$\begin{aligned}
(Q^0_{\frac{1}{2}})^+ &= (Q^0_{\frac{1}{2}})^- - \Delta Q_a - \Delta Q_b - \Delta Q_c \\
(Q^0_{1\frac{1}{2}})^+ &= (Q^0_{1\frac{1}{2}})^- + \Delta Q_c - \Delta Q_e \\
(Q^0_{2\frac{1}{2}})^+ &= (Q^0_{2\frac{1}{2}})^- + \Delta Q_b + \Delta Q_d + \Delta Q_e \\
(Q^0_{3\frac{1}{2}})^+ &= (Q^0_{3\frac{1}{2}})^- + \Delta Q_a - \Delta Q_d
\end{aligned} \tag{18}$$

There operations would be performed on each of the three passes involving actual transfer of $\Delta\mathcal{V}$. On the averaging pass the q for the complete zone would be found by

$$q_{1\frac{1}{2}} = \frac{Q^0_{1\frac{1}{2}} + Q^2_{1\frac{1}{2}} + Q^4_{1\frac{1}{2}} + Q^6_{1\frac{1}{2}}}{\mathcal{V}^0_{1\frac{1}{2}} + \mathcal{V}^2_{1\frac{1}{2}} + \mathcal{V}^4_{1\frac{1}{2}} + \mathcal{V}^6_{1\frac{1}{2}}} \tag{19}$$

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